

# Academic Standards for Mathematics

*June 1, 2009 FINAL  
Elementary Standards  
Grades 3-8*



*Pennsylvania Department of Education*

*These standards are offered as a voluntary resource for Pennsylvania's schools and await action by the State Board of Education.*

ELEMENTARY STANDARDS  
Grades 3-8  
Mathematics

IV. TABLE OF CONTENTS

**Introduction.....V.**

**THE ACADEMIC STANDARDS**

**Number, Number Systems and Number Relationships...2.1.**

*Count and Compare Numbers*  
*Represent Numbers in Equivalent Forms*  
*Concepts of Numbers and Relationships*  
*Place Value*  
*Number Theory*  
*Concepts and Applications of Operations*

**Computation and Estimation.....2.2.**

*Fluency in Basic Facts*  
*Computation*  
*Evaluate Numerical Expressions*  
*Numerical Estimation*

**Measurement and Estimation.....2.3.**

*Concept of Measurement*  
*Units and Tools of Measurement*  
*Calculations*  
*Conversions*  
*Relations*  
*Measurement Estimation*

**Mathematical Reasoning and Connections.....2.4.**

*Reasoning*  
*Connections*

**Mathematical Problem Solving and Communication.....2.5.**

*Problem Solving*  
*Communication*

**Statistics and Data Analysis.....2.6.**

*Collection of Data*  
*Organization and Display of Data*  
*Numerical Summaries*  
*Statistical Comparisons*  
*Interpretation of Data*

**Probability and Predictions.....2.7.**

*Calculate Probabilities*  
*Prediction of Outcomes*  
*Representations of Probabilities*  
*Display Simple Spaces*  
*Compare Theoretical and Experimental Probabilities*

**Algebra and Functions.....2.8.**

*Algebraic Properties*  
*Algebraic Manipulations*  
*Patterns*  
*Functions*  
*Modeling*  
*Interpret Results of Modeling*

**Geometry.....2.9.**

*Definitions, Properties, and Relations*  
*Transformations and Symmetry*  
*Coordinate Geometry*

**Trigonometry.....2.10.**

*Right Triangle Concepts and Applications*  
*Trigonometric Functions*

**Concepts of Calculus.....2.11.**

*Extreme Values*  
*Rates*  
*Accumulation of Areas and Volumes*

**Glossary.....VI.**

# ELEMENTARY STANDARDS

## Grades 3-8

## Mathematics

### V. INTRODUCTION

This document includes Mathematics Standards:

- |  |                                     |
|--|-------------------------------------|
| ◇ 2.1. Numbers, Number Systems, and Number Relationships | ◇ 2.6. Statistics and Data Analysis |
| ◇ 2.2. Computation and Estimation                        | ◇ 2.7. Probability and Predictions  |
| ◇ 2.3. Measurement and Estimation                        | ◇ 2.8. Algebra and Functions        |
| ◇ 2.4. Mathematical Reasoning and Connections            | ◇ 2.9. Geometry                     |
| ◇ 2.5. Mathematical Problem Solving and Communication    | ◇ 2.10. Trigonometry                |
|  | ◇ 2.11. Concepts of Calculus        |

The Mathematics Standards describe what students should know and be able to do at grade levels. They reflect the increasing complexity and sophistication that students are expected to achieve as they progress through school.

With each standard divided into conceptual strands, this document avoids repetition of learned skills and makes an obvious progression across grade levels less explicit. Teachers shall expect that students know and can apply the concepts and skills expressed at the preceding level. Consequently, previous learning is reinforced but not re-taught.

Students who achieve these mathematical standards will be able to communicate mathematically. Although it is an interesting and enjoyable study for its own sake, mathematics is most appropriately used as a tool to help organize and understand information from other academic disciplines. Because our capacity to deal with all things mathematical is changing rapidly, students must be able to bring the most modern and effective technology to bear on their learning of mathematical concepts and skills.

A glossary is included to assist the reader in understanding terminology contained in the standards. Words in bold faced text are included in the glossary.

# ELEMENTARY STANDARDS

## Grades 3-8

### Mathematics

2.1. Numbers, Number Systems and Number Relationships						
	2.1.3. GRADE 3	2.1.4. GRADE 4	2.1.5. GRADE 5	2.1.6. GRADE 6	2.1.7. GRADE 7	2.1.8. GRADE 8
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>						
Count and Compare Numbers	2.1.3.A. Apply <b>one-to-one correspondence</b> and number <b>patterns</b> to count up and count back and to compare values of whole numbers and values of money.	2.1.4.A. Apply number <b>patterns</b> and relationships to count and compare values of whole numbers and simple fractions, and decimals.	2.1.5.A. Apply number <b>patterns</b> to count and compare values of whole numbers, fractions, and decimals.	2.1.6.A. Model and compare values of whole numbers, mixed numbers, fractions, and decimals.	2.1.7.A. Model and compare values of <b>integers</b> , mixed numbers, fractions, and decimals.	2.1.8.A. Model and compare values of <b>integers</b> and <b>rational numbers</b> .
Represent Numbers in Equivalent Forms	2.1.3.B. Represent <b>equivalent forms</b> of the same number through the use of concrete objects, drawings, word names, and symbols.	2.1.4.B. Represent <b>equivalent forms</b> of the same whole number, the same fraction, or the same decimal through the use of concrete objects, drawings, word names, and symbols.	2.1.5.B. Use number theory concepts and <b>models</b> to represent or rename whole numbers, fractions, and decimals.	2.1.6.B. Represent whole numbers, fractions, mixed numbers, decimals, and percents in <b>equivalent</b> forms.	2.1.7.B. Represent and use numbers in <b>equivalent</b> forms (e.g. <b>integers</b> , fractions, decimals, percents, <b>exponents</b> , <b>powers</b> , <b>roots</b> , <b>absolute values</b> ).	2.1.8.B. Represent and use numbers in <b>equivalent</b> forms (e.g., <b>integers</b> , fractions, decimals, percents, <b>exponents</b> , <b>scientific notation</b> , square <b>roots</b> , <b>absolute values</b> ).
Concepts of Numbers and Relationships	2.1.3.C. Use drawings, diagrams or <b>models</b> to show the concept of fraction as part of a whole.	2.1.4.C. Use drawings, diagrams, or <b>models</b> to show the concept of a fraction as a part of a set and as division of a whole number by a whole number.	2.1.5.C. Use <b>models</b> to represent the concept of an <b>integer</b> , fraction, decimal, or percent.	2.1.6.C. Use <b>models</b> to represent the concept of <b>equivalent</b> forms of a fraction, decimal, and/or percent.	2.1.7.C. Use <b>ratio</b> and <b>proportion</b> to model relationships between quantities.	2.1.8.C. Use <b>ratio</b> and <b>proportion</b> to model relationships between quantities.

# ELEMENTARY STANDARDS

## Grades 3-8

### Mathematics

2.1. Numbers, Number Systems and Number Relationships						
	2.1.3. GRADE 3	2.1.4. GRADE 4	2.1.5. GRADE 5	2.1.6. GRADE 6	2.1.7. GRADE 7	2.1.8. GRADE 8
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>						
Place Value	2.1.3.D. Apply place value concepts and base-ten numeration to order and compare whole numbers.	2.1.4.D. Apply place value concepts and base-ten numeration to order and compare larger whole numbers.	2.1.5.D. Apply place value concepts to order and compare decimals and to express whole numbers and decimals in <b>expanded notation</b> .	2.1.6.D. Apply place value concepts to order and compare decimals; use the number line to order and compare decimals, fractions, and mixed numbers.	2.1.7.D. Apply place value concepts to order and compare decimals; use the number line to order and compare decimals, fractions, mixed numbers, and/or <b>integers</b> .	2.1.8.D. Extend place value concepts to represent large numbers using <b>exponential, scientific,</b> and calculator notation.
Number Theory	2.1.3.E. Apply number <b>patterns</b> even and odd, <b>factors</b> and <b>multiples</b> to represent numbers in various ways.	2.1.4.E. Apply <b>factors</b> and <b>multiples</b> to represent larger numbers in various ways.	2.1.5.E. Develop and apply number theory concepts (e.g., <b>primes, factors, multiples, composites</b> ) to represent numbers in various ways.	2.1.6.E. Apply number theory concepts to calculate the GCF (Greatest Common Factor) and/or LCM (Least Common Multiple) of two numbers.	2.1.7.E. Apply concepts of <b>prime</b> and <b>composite</b> numbers to calculate GCFs (Greatest Common Factor) and LCMs (Least Common Multiple) of numbers.	2.1.8.E. Apply concepts of <b>prime</b> and <b>composite</b> numbers to calculate GCFs (Greatest Common Factor) and LCMs (Least Common Multiple) of numbers.
Concepts and Applications of Operations	2.1.3.F. Understand the concepts of addition and subtraction and use the <b>inverse</b> relationships between addition and subtraction to determine unknown quantities in <b>equations</b> .	2.1.4.F. Understand the concepts of addition and subtraction and their <b>inverse</b> relationships; understand the concepts of multiplication and division; use the four basic operations to solve problems, including word problems and <b>equations</b> .	2.1.5.F. Understand the concepts of multiplication and division and use the <b>inverse</b> relationships between multiplication and division, to determine unknown quantities in <b>equations</b> .	2.1.6.F. Apply the <b>associative, commutative, distributive</b> and/or <b>identity</b> properties to write <b>equivalent</b> forms of <b>expressions</b> .	2.1.7.F. Understand the concepts of <b>ratio, proportion, percents, and rates</b> to determine unknown quantities in <b>equations</b> .	2.1.8.F. Understand the concepts of <b>exponents</b> and <b>roots</b> and use the <b>inverse</b> relationships between <b>exponents</b> and <b>roots</b> to determine unknown quantities in <b>equations</b> .

# ELEMENTARY STANDARDS

## Grades 3-8

### Mathematics

2.2. Computation and Estimation						
	2.2.3. GRADE 3	2.2.4. GRADE 4	2.2.5. GRADE 5	2.2.6. GRADE 6	2.2.7. GRADE 7	2.2.8. GRADE 8
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>						
Fluency in Basic Facts	2.2.3.A. Develop <b>fluency</b> in the use of basic facts for the four operations.	2.2.4.A. Develop <b>fluency</b> in the use of basic facts for the four <b>operations</b> .	2.2.5.A. Intentionally Blank	2.2.6.A. Intentionally Blank	2.2.7.A. Intentionally Blank	2.2.8.A. Intentionally Blank
Computation	2.2.3.B. Add and subtract single- and double-digit numbers with regrouping and triple-digit numbers, without regrouping including problems with money.	2.2.4.B. Multiply single- and double-digit numbers and divide by single digit numbers, add and subtract fractions with like denominators, and add and subtract decimals.	2.2.5.B. Multiply and divide single- and double-digit numbers; add and subtract fractions and mixed numbers; add, subtract, multiply and divide decimals.	2.2.6.B. Add, subtract, multiply, and divide whole numbers, decimals, fractions, and mixed numbers.	2.2.7.B. Add, subtract, multiply, and divide whole numbers, decimals, fractions, mixed numbers, or <b>integers</b> .	2.2.8.B. Add, subtract, multiply, and divide different kinds and forms of <b>rational numbers</b> including <b>integers</b> , decimal fractions, percents, and proper and improper fractions.
Evaluate Numerical Expressions	2.2.3.C. Intentionally Blank	2.2.4.C. Intentionally Blank	2.2.5.C. Intentionally Blank	2.2.6.C. Apply the associative, commutative, distributive, and/or identity properties to evaluate numerical <b>expressions</b> .	2.2.7.C. Use the <b>order of operations</b> to evaluate numerical <b>expressions</b> .	2.2.8.C. Use the <b>order of operations</b> to evaluate numerical <b>expressions</b> .

# ELEMENTARY STANDARDS

## Grades 3-8

### Mathematics

2.2. Computation and Estimation						
2.2.3. GRADE 3		2.2.4. GRADE 4	2.2.5. GRADE 5	2.2.6. GRADE 6	2.2.7. GRADE 7	2.2.8. GRADE 8
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>						
Numerical Estimation	2.2.3.D. Estimate values, sums, and differences of quantities and conclude the <b>reasonableness</b> of those estimates.	2.2.4.D. Estimate sums and differences, products, and quotients, and conclude the <b>reasonableness</b> of those estimates.	2.2.5.D. Estimate results from calculations with basic operations of whole numbers and decimals and check the <b>reasonableness</b> of those estimates.	2.2.6.D. Estimate solutions of problems involving whole numbers and decimals and check the <b>reasonableness</b> of those estimates.	2.2.7.D. Estimate solutions of problems involving calculations with basic operations of whole numbers, decimals, fractions, or mixed numbers and check the <b>reasonableness</b> of those estimates.	2.2.8.D. Estimate the values of <b>irrational numbers</b> and the results from calculations with basic operations of fractions and percents and check the <b>reasonableness</b> of those estimates.

# ELEMENTARY STANDARDS

## Grades 3-8

### Mathematics

<b>2.3. Measurement and Estimation</b>						
	<b>2.3.3. GRADE 3</b>	<b>2.3.4. GRADE 4</b>	<b>2.3.5. GRADE 5</b>	<b>2.3.6. GRADE 6</b>	<b>2.3.7. GRADE 7</b>	<b>2.3.8. GRADE 8</b>
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>						
<b>Concept of Measurement</b>	2.3.3.A. Demonstrate an understanding of measurable characteristics and the need to quantify those characteristics.	2.3.4.A. Use concrete objects to demonstrate an understanding of measurement quantities (e.g., length, weight, temperature).	2.3.5.A. Use concrete objects to demonstrate the meaning of measurement quantities (e.g., perimeter, area, weight, capacity).	2.3.6.A. Use <b>models</b> to illustrate the meaning of perimeter, area, and <b>volume</b> .	2.3.7.A. Demonstrate an understanding of measurable attributes and the units, systems, and processes of measurement.	2.3.8.A. Intentionally Blank
<b>Units and Tools of Measurement</b>	2.3.3.B. Identify a measurable characteristic of an object, select an appropriate standard or non-standard unit of measure and tool, and determine the measurement to a specified level of accuracy.	2.3.4.B. Select and use appropriate tools and units for measuring quantities (e.g., length, time, weight, temperature).	2.3.5.B. Select and use appropriate instruments and units for measuring quantities to a specified level of accuracy.	2.3.6.B. Use appropriate units to measure perimeter, area, and <b>volume</b> ; use a protractor to measure angles between 0 and 180 degrees.	2.3.7.B. Develop strategies for and use appropriate units to determine lengths, areas, and perimeters of compound shapes.	2.3.8.B. Develop strategies for determining areas and <b>volumes</b> of compound shapes and solids.
<b>Calculations</b>	2.3.3.C. Tell time on an <b>analog</b> and digital clock, identify times of day and night as a.m. and p.m., and calculate elapsed time.	2.3.4.C. Calculate elapsed time; use concept of elapsed time to determine start time/end time.	2.3.5.C. Calculate perimeter and area, and sums and differences of measurements.	2.3.6.C. Use given measurements to calculate a missing length, perimeter, area, and/or <b>volume</b> ; Calculate elapsed time across am/pm and across days.	2.3.7.C. Use measurement formulas to calculate <b>volume</b> , area, and perimeter and to calculate circumference and area of circles.	2.3.8.C. Calculate <b>volume</b> , surface area, and degrees of angles; calculate circumference and area of circles, and use a measurement formula to solve for a missing quantity.



# ELEMENTARY STANDARDS

## Grades 3-8

### Mathematics

2.3. Measurement and Estimation						
	2.3.3. GRADE 3	2.3.4. GRADE 4	2.3.5. GRADE 5	2.3.6. GRADE 6	2.3.7. GRADE 7	2.3.8. GRADE 8
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>						
<b>Conversions</b>	2.3.3.D. Identify <b>equivalent</b> measurements within the same system.	2.3.4.D. Perform basic conversions within the same system to the unit immediately above or below the given unit.	2.3.5.D. Perform basic conversions within a system.	2.3.6.D. Perform basic conversions within the <b>metric</b> and within the <b>customary systems</b> .	2.3.7.D. Use conversions to add and subtract measurement quantities within the <b>metric</b> and within the <b>customary systems</b> .	2.3.8.D. Perform conversions within the <b>metric system</b> and within the <b>customary system</b> including scale measurements, between units of time and between units of temperature.
<b>Relations</b>	2.3.3.E. Intentionally Blank	2.3.4.E. Intentionally Blank	2.3.5.E. Intentionally Blank	2.3.6.E. Intentionally Blank	2.3.7.E. Select and/or use an appropriate scale for creating enlarged or reduced representations.	2.3.8.E. Describe how a change in linear dimension of an object affects its perimeter, area and volume.
<b>Measurement Estimation</b>	2.3.3.F. Estimate and verify measurements of length, area, weight, and capacity.	2.3.4.F. Estimate and verify measurements of length, perimeter, area, weight, capacity, temperature, and time.	2.3.5.F. Estimate and verify measurements of length, perimeter, area, <b>volume</b> , capacity, temperature, time, weight, and angles.	2.3.6.F. Estimate and verify measurements of length, perimeter, area, <b>volume</b> , capacity, temperature, time, weight, and angles.	2.3.7.F. Estimate and verify measurements of length, perimeter, area, <b>volume</b> , capacity, temperature, time, weight, and angles.	2.3.8.F. Estimate and verify measurements of rate and mass.

# ELEMENTARY STANDARDS

## Grades 3-8

### Mathematics

2.4. Mathematical Reasoning and Connections						
	2.4.3. GRADE 3	2.4.4. GRADE 4	2.4.5. GRADE 5	2.4.6. GRADE 6	2.4.7. GRADE 7	2.4.8. GRADE 8
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>						
<b>Reasoning</b>	2.4.3.A. Use <b>models</b> and number facts to draw conclusions and explain reasons for conclusions.	2.4.4.A. Use <b>models</b> , number facts, and properties to make conjectures, draw conclusions and explain reasons for conclusions.	2.4.5.A. Use <b>models</b> , number facts, properties and relationships to draw conclusions and explain reasons for conclusions.	2.4.6.A Use <b>models</b> , properties, and relationships to draw conclusions and explain reasons for conclusions.	2.4.7.A. Draw logical conclusions and justify reasoning for conclusions within mathematical contexts.	2.4.8.A. Draw <b>inductive</b> and <b>deductive</b> conclusions within mathematical contexts.
<b>Connections</b>	2.4.3.B. Interpret statements made with precise language of logic (e.g., <i>all, or, every, none, some, or, many.</i> )	2.4.4.B. Recognize and use precise language to describe connections between mathematical ideas.	2.4.5.B. Use <i>if...then</i> statements to express conditional relationships.	2.4.6.B. Use <i>if...then</i> statements to express conditional relationships.	2.4.7.B. Evaluate the truth of conditional relationships expressed as <i>if...then</i> statements.	2.4.8.B. Use <i>if...then</i> statements to construct simple <b>valid arguments</b> .

# ELEMENTARY STANDARDS

## Grades 3-8

### Mathematics

<b>2.5. Mathematical Problem Solving and Communication</b>						
	<b>2.5.3. GRADE 3</b>	<b>2.5.4. GRADE 4</b>	<b>2.5.5. GRADE 5</b>	<b>2.5.6. GRADE 6</b>	<b>2.5.7. GRADE 7</b>	<b>2.5.8. GRADE 8</b>
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>						
<b>Problem Solving</b>	2.5.3.A. Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, check whether an answer makes sense, and explain how the problem was solved in grade appropriate contexts.	2.5.4.A. Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, check whether an answer makes sense, and explain how the problem was solved in grade appropriate contexts.	2.5.5.A. Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, check whether an answer makes sense, and explain how the problem was solved in grade appropriate contexts.	2.5.6.A. Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, check whether an answer makes sense, and explain how the problem was solved in grade appropriate contexts.	2.5.7.A. Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, apply estimation skills as appropriate, check whether the plan makes sense, and explain how the problem was solved in grade appropriate contexts.	2.5.8.A. Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, apply estimation skills as appropriate, check whether the plan makes sense, and explain how the problem was solved in grade appropriate contexts.
<b>Communication</b>	2.5.3.B. Use appropriate mathematical vocabulary when explaining how to solve a problem.	2.5.4.B. Use appropriate mathematical vocabulary, graphs, and symbols when explaining how to solve a problem.	2.5.5.B. Use appropriate mathematical terms, vocabulary, language, symbols, and graphs to explain clearly and logically solutions to problems.	2.5.6.B. Use appropriate mathematical terms, vocabulary, language, symbols, and graphs to explain clearly and logically solutions to problems.	2.5.7.B. Use appropriate mathematical language, notation, and representations, including numerical tables, <b>expressions</b> , and <b>equations</b> ; including formulas, charts, graphs, and diagrams to explain and interpret results.	2.5.8.B. Use precise mathematical language, notation, and representations, including numerical tables and <b>equations</b> , simple algebraic <b>equations</b> , and formulas, charts, graphs and diagrams to explain and interpret results.

# ELEMENTARY STANDARDS

## Grades 3-8

### Mathematics

2.6. Statistics and Data Analysis						
	2.6.3. GRADE 3	2.6.4. GRADE 4	2.6.5. GRADE 5	2.6.6. GRADE 6	2.6.7. GRADE 7	2.6.8. GRADE 8
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>						
Collection of Data	2.6.3.A. Gather data from surveys and observations within the classroom or homes.	2.6.4.A. Gather data from surveys and observations within the classroom or homes.	2.6.5.A. Gather data from surveys and observations from sources outside the classroom or home.	2.6.6.A. Gather data from a variety of appropriate sources.	2.6.7.A. Identify different ways of selecting a sample and choosing an appropriate sampling technique for a given situation.	2.6.8.A. Understand and apply sampling techniques to gather data including simple random sampling and convenience sampling.
Organization and Display of Data	2.6.3.B. Organize and display data using pictures, tallies, charts, bar graphs and pictographs.	2.6.4.B. Organize and display data using tables, pictures, tallies, bar graphs, line graphs, or pictographs.	2.6.5.B. Use pictures, tallies, tables, charts, bar graphs, line graphs, diagrams, and graphs to organize, display, and analyze data.	2.6.6.B. Select an appropriate method to organize data; select an appropriate format to display data.	2.6.7.B. Organize and display data using an appropriate data display, such as circle graphs, <b>histograms</b> , line graphs, double bar graphs, and <b>stem-and-leaf plots</b> , <b>Venn diagrams</b> , tables, and charts.	2.6.8.B. Organize and display one- <b>variable</b> data using appropriate data display, such as <b>stem-and-leaf</b> and <b>box-and-whisker plots</b> , and two <b>variable</b> data with <b>scatterplots</b> .
Numerical Summaries	2.6.3.C. Describe data displayed in a diagram (e.g., <b>Venn</b> ) a graph or a table.	2.6.4.C. Describe and calculate the <b>mean</b> and use this quantity to describe the data.	2.6.5.C. Calculate <b>mean</b> and <b>range</b> , identify the <b>median</b> and the <b>mode</b> of a set of data, and use these quantities to describe the data.	2.6.6.C Select and use, as appropriate, the <b>mean</b> , <b>median</b> , <b>mode</b> , and/or <b>range</b> to describe sets of data.	2.6.7.C. Use numerical summaries to describe different sets of data.	2.6.8.C. Calculate <b>quartiles</b> for one- <b>variable</b> data and describe the <b>correlation</b> coefficient for two- <b>variable</b> data displayed in a <b>scatterplot</b> .

# ELEMENTARY STANDARDS

## Grades 3-8

### Mathematics

2.6. Statistics and Data Analysis						
2.6.3. GRADE 3		2.6.4. GRADE 4	2.6.5. GRADE 5	2.6.6. GRADE 6	2.6.7. GRADE 7	2.6.8. GRADE 8
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>						
Statistical Comparisons	2.6.3.D. Analyze data shown in tables, charts, diagrams, and graphs; compare the data from two categories displayed in a graph and compare representations of a set of data in different graphs.	2.6.4.D. Analyze data shown in tables, charts, diagrams, and graphs; compare the data from two categories displayed in a graph and compare representations of a set of data in different graphs.	2.6.5.D. Compare data using multiple categories displayed in a graph.	2.6.6.D Use measures of <b>central tendency</b> to compare two sets of data.	2.6.7.D. Use measures of <b>central tendency</b> and spread to compare data sets.	2.6.8.D. Compare data sets graphically using double-bar and double-line graphs and numerically using <b>mean, median, mode, range, and quartiles.</b>
Interpretation of Data	2.6.3.E. Determine the <b>reasonableness</b> of a statement based on a comparison to data displayed in a graph.	2.6.4.E. Determine the <b>reasonableness</b> of a statement based on a comparison to data displayed in a graph.	2.6.5.E. Determine the <b>reasonableness</b> of a statement based on a comparison to data displayed in a graph and summarized by numerical measures.	2.6.6.E. Interpret data displayed in a table, <b>histogram</b> , graph, or data summarized by numerical measures.	2.6.7.E. Interpret trends and make predictions based on data displayed in a graph.	2.6.8.E. Determine the effect of extreme values on numerical summaries and calculate estimates based on survey results or graphs.

# ELEMENTARY STANDARDS

## Grades 3-8

### Mathematics

<b>2.7. Probability and Predictions</b>						
	<b>2.7.3. GRADE 3</b>	<b>2.7.4. GRADE 4</b>	<b>2.7.5. GRADE 5</b>	<b>2.7.6. GRADE 6</b>	<b>2.7.7. GRADE 7</b>	<b>2.7.8. GRADE 8</b>
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>						
<b>Calculation of Probabilities</b>	2.7.3.A. Determine the chance of an event occurring by performing <b>simulations</b> with concrete devices (e.g., dice, spinner).	2.7.4.A. Determine the chance of an event occurring by performing <b>simulations</b> with concrete devices (e.g., dice, spinner).	2.7.5.A. Predict and calculate the likelihood of <b>simple events</b> .	2.7.6.A. Collect data and estimate the likelihood of outcomes of an event.	2.7.7.A. Predict the outcome of a grade-level appropriate <b>probability</b> experiment.	2.7.8.A. Calculate the <b>probability</b> of an event involving “and”, “or” or “not”.
<b>Prediction of Outcomes</b>	2.7.3.B. Determine whether different outcomes of the same event are equally likely or not equally likely.	2.7.4.B. Determine whether different outcomes of the same event are equally likely or not equally likely.	2.7.5.B. Predict and determine why some outcomes of a particular event are certain, more likely, less likely, equally likely, or impossible.	2.7.6.B. Organize data collected in a simulation and select an appropriate format to display the data.	2.7.7.B. Organize data collected in an experiment and select an appropriate format to display the data.	2.7.8.B. Intentionally Blank
<b>Representations of Probabilities</b>	2.7.3.C. Write the likelihood of an event as a fraction.	2.7.4.C. Express <b>probabilities</b> as fractions.	2.7.5.C. Express <b>probabilities</b> as fractions and/or decimals.	2.7.6.C. Express the <b>probability</b> of a simple event as a fraction, decimal, and percent.	2.7.7.C. Express the <b>probability</b> of a <b>compound</b> or <b>complimentary event</b> as a fraction, decimal, or percent.	2.7.8.C. Determine the number of <b>combinations</b> and <b>permutations</b> for an event.
<b>Display Simple Spaces</b>	2.7.3.D. List or graph the possible results of an experiment.	2.7.4.D. List or graph the possible results of an experiment.	2.7.5.D. List all possible <b>combinations</b> and arrangements of outcomes of an experiment (e.g., <b>tree diagrams</b> , <b>matrices</b> )	2.7.6.D. List the possible outcomes for two <b>independent events</b> and compare the outcomes.	2.7.7.D. List the possible outcomes for two or more <b>independent events</b> and compare the outcomes.	2.7.8.D. Intentionally Blank

ELEMENTARY STANDARDS  
Grades 3-8  
Mathematics

2.7. Probability and Predictions						
2.7.3. GRADE 3		2.7.4. GRADE 4	2.7.5. GRADE 5	2.7.6. GRADE 6	2.7.7. GRADE 7	2.7.8. GRADE 8
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>						
Compare Theoretical and Experimental Probabilities	2.7.3.E. Determine that there can be a difference between predicted and actual outcomes.	2.7.4.E. Describe possible reasons for the difference between predicted and actual outcomes.	2.7.5.E. Compare predictions based on theoretical <b>probability</b> and experimental results.	2.7.6.E. Find and interpret the experimental <b>probability</b> of an outcome of a simple event.	2.7.7.E. Find and interpret the experimental or theoretical <b>probability</b> of an outcome of a simple event.	2.7.8.E. Find the experimental or theoretical <b>probability</b> of the outcomes of a simple or <b>compound event</b> .

# ELEMENTARY STANDARDS

## Grades 3-8

### Mathematics

2.8. Algebra and Functions						
	2.8.3. GRADE 3	2.8.4. GRADE 4	2.8.5. GRADE 5	2.8.6. GRADE 6	2.8.7. GRADE 7	2.8.8. GRADE 8
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>						
Algebraic Properties	2.8.3.A. Use the concept of equality and concrete objects to demonstrate understanding of <b>commutative, associative, and identity</b> properties.	2.8.4.A. Use the concept of equality and concrete objects to demonstrate understanding of <b>commutative, associative, and identity</b> properties.	2.8.5.A. Use the concept of equality to demonstrate understanding of the <b>distributive</b> property.	2.8.6.A. Use the concept of equality to demonstrate understanding of the <b>distributive</b> property.	2.8.7.A. Use the concept of equality to demonstrate understanding of properties applied to <b>rational numbers</b> (e.g. <b>identity, distributive, associative, commutative</b> ).	2.8.8.A. Use the concept of equality to demonstrate understanding of the <b>inverse</b> properties of numbers and the addition and multiplication properties of equality.
Algebraic Manipulations	2.8.3.B. Use concrete objects and trial and error to solve number sentences ( <b>equations</b> and <b>inequalities</b> ).	2.8.4.B. Select and use strategies, including concrete objects, to solve number sentences ( <b>equations</b> and <b>inequalities</b> ) involving whole numbers or unit fractions and explain the method of solution.	2.8.5.B. Select and use strategies, including concrete objects, to solve number sentences ( <b>equations</b> and <b>inequalities</b> ) and explain the method of solution.	2.8.6.B. Select and use strategies to solve number sentences (and <b>inequalities</b> ) and explain the method of solution.	2.8.7.B. Evaluate and simplify algebraic <b>expressions</b> and solve and graph linear <b>equations</b> and <b>inequalities</b> .	2.8.8.B. Evaluate and simplify algebraic <b>expressions</b> and solve and graph linear <b>equations</b> and <b>inequalities</b> .



# ELEMENTARY STANDARDS

## Grades 3-8

### Mathematics

2.8. Algebra and Functions						
	2.8.3. GRADE 3	2.8.4. GRADE 4	2.8.5. GRADE 5	2.8.6. GRADE 6	2.8.7. GRADE 7	2.8.8. GRADE 8
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>						
<b>Patterns</b>	2.8.3.C. Recognize, describe, extend, create, and replicate a variety of <b>patterns</b> including attribute, activity, number, and geometric <b>patterns</b> .	2.8.4.C. Recognize, describe, extend, create, replicate, and make generalizations for a variety of <b>patterns, sequences</b> , and relationships verbally and numerically.	2.8.5.C. Recognize, describe, extend, create, replicate, and form a rule for a variety of <b>patterns, sequences</b> , and relationships verbally, numerically, symbolically, and graphically.	2.8.6.C. Recognize, describe, extend, create, replicate, form a rule, and/or find a missing element for a variety of whole number <b>patterns, sequences</b> , and relationships verbally, numerically, symbolically, and graphically.	2.8.7.C. Recognize, describe, extend, create, replicate, form a rule, and/or find a missing element for a variety of <b>rational number patterns, sequences</b> , and relationships verbally, numerically, symbolically, and graphically.	2.8.8.C. Find the missing elements and recognize, describe, and extend <b>patterns</b> to include <b>linear, exponential</b> , and simple <b>quadratic equations</b> .
<b>Functions</b>	2.8.3.D. Use a rule to find a missing value and determine a rule for a given <b>pattern</b> .	2.8.4.D. Use words, tables, and graphs to represent and analyze <b>functions</b> .	2.8.5.D. Determine a functional rule from a table or graph.	2.8.6.D. Determine a functional rule from a table or graph.	2.8.7.D. Determine a functional rule from given data or a situation.	2.8.8.D. Create a table or graph from a functional rule.
<b>Modeling</b>	2.8.3.E. Use concrete objects or combinations of symbols and numbers to represent <b>expressions, equations</b> , and <b>inequalities</b> that model mathematical situations.	2.8.4.E. Use concrete objects and combinations of symbols and numbers to create <b>expressions, equations</b> , and <b>inequalities</b> that model mathematical situations.	2.8.5.E. Use concrete objects and combinations of symbols and numbers to create <b>expressions, equations</b> , and <b>inequalities</b> that model mathematical situations.	2.8.6.E. Use combinations of symbols and numbers to create <b>expressions, equations</b> , and <b>inequalities</b> that model mathematical situations.	2.8.7.E. Use combinations of symbols and numbers to create <b>expressions, equations</b> , and <b>inequalities</b> in one <b>variable</b> that model problem situations.	2.8.8.E. Use combinations of symbols and numbers to create <b>expressions</b> and <b>equations</b> in one or two <b>variables</b> , and <b>inequalities</b> in one <b>variable</b> that model problem situations.

ELEMENTARY STANDARDS  
Grades 3-8  
Mathematics

2.8. Algebra and Functions						
2.8.3. GRADE 3		2.8.4. GRADE 4	2.8.5. GRADE 5	2.8.6. GRADE 6	2.8.7. GRADE 7	2.8.8. GRADE 8
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>						
Interpret Results of Modeling	2.8.3.F. Describe data represented in a table, chart, or number sentence and/or create a story that matches that data.	2.8.4.F. Describe data represented in <b>equations, inequalities</b> , tables, or graphs and/or create a story that matches that data.	2.8.5.F. Describe data represented in <b>equations, inequalities</b> , tables, or graphs and/or create a story that matches that data.	2.8.6.F. Interpret the results of solving <b>equations</b> in one <b>variable</b> in the context of the situation that motivated the model.	2.8.7.F. Interpret the results of solving <b>equations and inequalities</b> in one <b>variable</b> in the context of the situation that motivated the model.	2.8.8.F. Interpret the results of solving <b>equations</b> in one or two <b>variables</b> and <b>inequalities</b> in one <b>variable</b> in the context of the situation that motivated the model.

# ELEMENTARY STANDARDS

## Grades 3-8

### Mathematics

2.9. Geometry						
	2.9.3. GRADE 3	2.9.4. GRADE 4	2.9.5. GRADE 5	2.9.6. GRADE 6	2.9.7. GRADE 7	2.9.8. GRADE 8
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>						
Definitions, Properties, and Relations	2.9.3.A. Name, describe and draw/build 2- and 3-dimensional shapes.	2.9.4.A. Identify, describe, and define 1-, 2-, and 3-dimensional shapes and their related parts; compare 2-dimensional shapes; compare 3-dimensional shapes.	2.9.5.A. Identify, describe, and define 1-, 2-, and 3-dimensional shapes and their related parts, and classify and compare 2- and 3-dimensional shapes on the basis of their properties.	2.9.6.A. Identify, define, label, and/or describe properties of 1-, 2-, and 3-dimensional shapes and their related parts, and classify and compare 2- and 3-dimensional shapes on the basis of their properties.	2.9.7.A. Identify, define, label, and/or describe properties of 1-, 2-, and 3-dimensional shapes and their related parts, and classify and compare 2- and 3-dimensional shapes on the basis of their properties.	2.9.8.A. Name, describe and apply geometric relations for 1-dimensional shapes and 2-dimensional shapes and 3-dimensional solids.
Transformations and Symmetry	2.9.3.B. Identify and draw lines of <b>symmetry</b> .	2.9.4.B. Identify and draw figures with one or more lines of <b>symmetry</b> .	2.9.5.B. Predict and describe the result of a <b>translation</b> (slide), <b>rotation</b> (turn), or <b>reflection</b> (flip) of a 2-dimensional shape.	2.9.6.B. Predict and describe the result of a <b>translation</b> (slide), <b>rotation</b> (turn), or <b>reflection</b> (flip) of a 2-dimensional shape.	2.9.7.B. Predict and describe the result of a <b>translation</b> (slide), <b>rotation</b> (turn), and/or <b>reflection</b> (flip) of a 2-dimensional shape.	2.9.8.B. Predict and describe the result of a <b>translation</b> (slide), <b>rotation</b> (turn), or <b>reflection</b> (flip) of a 3-dimensional shape.
Coordinate Geometry	2.9.3.C. Identify locations of points with whole number coordinates on a number line or on a 2-dimensional <b>coordinate system</b> .	2.9.4.C. Identify on a 2-dimensional <b>coordinate system</b> the location of points with whole number coordinates; plot in a two-dimensional coordinate system a point represented by an ordered pair of whole numbers	2.9.5.C. Identify location of points with fractional or decimal coordinates on a number line or on a 2-dimensional <b>coordinate system</b> .	2.9.6.C. Identify on a 2-dimensional <b>coordinate system</b> the location of points with non-negative fractional or decimal coordinates; plot in a two-dimensional <b>coordinate system</b> a point represented by an ordered pair of non-negative fractions, mixed numbers, or decimals.	2.9.7.C. Identify on a 2-dimensional <b>coordinate system</b> the location of points with <b>rational number</b> coordinates; plot in a two-dimensional <b>coordinate system</b> a point represented by an ordered pair of <b>rational numbers</b> .	2.9.8.C. Plot ordered pairs and 2-dimensional shapes that satisfy given conditions on a 2-dimensional <b>coordinate system</b> .

# ELEMENTARY STANDARDS

## Grades 3-8

### Mathematics

<b>2.10. Trigonometry</b>						
	<b>2.10.3. GRADE 3</b>	<b>2.10.4. GRADE 4</b>	<b>2.10.5. GRADE 5</b>	<b>2.10.6. GRADE 6</b>	<b>2.10.7. GRADE 7</b>	<b>2.10.8. GRADE 8</b>
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>						
<b>Right Triangle Concepts and Applications</b>	2.10.3.A. Identify right angles in the environment.	2.10.4.A. Identify right angles in geometric figures.	2.10.5.A. Identify and compare parts of right triangles, including right angles, acute angles, hypotenuses, and legs.	2.10.6.A. Identify and compare parts of right triangles, including right angles, acute angles, hypotenuses, and legs.	2.10.7.A. Compute measures of sides and angles using <b>proportions</b> , the Pythagorean Theorem, and right triangle relationships.	2.10.8.A. Compute measures of sides and angles using <b>proportions</b> , the Pythagorean Theorem, and right triangle relationships.
<b>Trigonometric Functions</b>	2.10.3.B. Intentionally Blank	2.10.4.B. Intentionally Blank	2.10.5.B. Intentionally Blank	2.10.6.B. Intentionally Blank	2.10.7.B. Intentionally Blank	2.10.8.B. Intentionally Blank

# ELEMENTARY STANDARDS

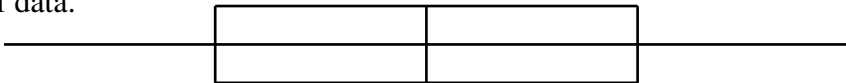
## Grades 3-8

### Mathematics

2.11. Concepts of Calculus						
	2.11.3. GRADE 3	2.11.4. GRADE 4	2.11.5. GRADE 5	2.11.6. GRADE 6	2.11.7. GRADE 7	2.11.8. GRADE 8
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>						
<b>Extreme Values</b>	2.11.3.A. Identify whole number quantities and measurements from least to most and greatest value.	2.11.4.A. Make comparisons of whole numbers and of unit fractions (e.g., more, less, same, least, most, greater than, less than).	2.11.5.A. Make comparisons of numbers (e.g., more, less, same, least, most, greater than, less than).	2.11.6.A. Make comparisons of numbers (e.g., more, less, same, least, most, greater than, less than).	2.11.7.A. Compare and order <b>rational numbers</b> ; identify the maximum and/or minimum values of a set of numbers.	2.11.8.A. Analyze graphs of related quantities for minimum and maximum values and justify the findings.
<b>Rates</b>	2.11.3.B. Intentionally Blank	2.11.4.B. Intentionally Blank	2.11.5.B. Describe the relationship between <b>rates of change</b> and another <b>variable</b> (e.g., time, temperature).	2.11.6.B. Describe the relationship between <b>rates of change</b> and another <b>variable</b> (e.g., time, temperature).	2.11.7.B. Describe and use the relationship of data shown in a graph; solve problems involving a constant rate of change.	2.11.8.B. Describe the concept of unit rate, <b>ratio</b> , and <b>slope</b> in the context of rate of change.
<b>Accumulation of Areas and Volumes</b>	2.11.3.C. Intentionally Blank	2.11.4.C. Intentionally Blank	2.11.5.C. Estimate areas and volumes of shapes and solids as the sums of areas of tiles and volumes of cubes.	2.11.6.C. Estimate areas and volumes of shapes and solids as the sums of areas of tiles and volumes of cubes.	2.11.7.C. Intentionally Blank	2.11.8.C. Intentionally Blank

ELEMENTARY STANDARDS  
Grades 3-8  
Mathematics

**VI. GLOSSARY**

<b>Absolute Value:</b>	A number's distance from zero on a number line. The absolute value of 2 is equal to the absolute value of -2.
<b>Algorithm:</b>	A method of performing an arithmetic operation.
<b>Analog Time:</b>	Time displayed on a timepiece having hour and minute hands.
<b>Array:</b>	Arrangement of a series of items according to the values of the items (e.g., largest to smallest).
<b>Associative Property:</b>	A property of addition or multiplication in which the regrouping of the addends or factors does not change the outcome of the operations [i.e., $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$ ].
<b>Box-and-Whisker Plot:</b>	<div><p>A graphic method for showing a summary of data using median, quartiles and extremes of data.</p></div>
<b>Central Tendency:</b>	The degree of clustering of the values of a statistical distribution that is usually measured by the arithmetic mean, mode, or median.
<b>Combination:</b>	A subset of the elements in a given set, without regard to the order in which those elements are arranged.
<b>Commutative Property:</b>	A property of addition or multiplication in which the sum or product stays the same when the order of the addends or factors is changed (i.e., $a + b = b + a$ and $ab = ba$ ).
<b>Complementary Event:</b>	The opposite of an event. That is, the set of all outcomes of an experiment that are not included in an event. $P(A') = 1 - P(A)$ .

# ELEMENTARY STANDARDS

## Grades 3-8

### Mathematics

<b>Complex Numbers:</b>	Any number, real or imaginary, of the form $a + bi$ , where $a$ and $b$ are real numbers and $i = \sqrt{-1}$ .
<b>Composite Number:</b>	Any positive integer exactly divisible by one or more positive integers other than itself and 1.
<b>Compound Event:</b>	An event that consists of two or more simple events; for example: A or B; A and B and C.
<b>Congruent:</b>	Having the same shape and the same size.
<b>Conjecture:</b>	A statement believed to be true but not proved.
<b>Contrapositive:</b>	The implication which results from replacing the antecedent by the negation of the consequent and the consequent by the negation of the antecedent.
<b>Converse:</b>	The theorem (or implication) resulting from interchanging the hypothesis and conclusion.
<b>Coordinate System:</b>	A method of locating points in the plane or in space by means of numbers. A point in the plane is located by its distances from both a horizontal and a vertical line called the axes. The horizontal line is called the x-axis. The vertical line is called the y-axis. The pairs of numbers are called ordered pairs. The first number, called the x-coordinate, designates the distance along the horizontal axis. The second number, called the y-coordinate, designates the distance along the vertical axis. The point at which the two axes intersect has the coordinates (0,0) and is called the origin.
<b>Correlation :</b>	A measure of the mutual relationship between two variables.
<b>Customary System:</b>	A system of weights and measures frequently used in the United States. The basic unit of weight is the pound; the basic unit of capacity is the quart.
<b>Deductive Reasoning:</b>	The process of reasoning from statements accepted as true to reach a conclusion.
<b>Dependent Event:</b>	The probability of one event depends on the outcome of another event.

# ELEMENTARY STANDARDS

## Grades 3-8

### Mathematics

<b>Direct Proof:</b>	Uses an argument that makes direct use of the hypothesis and arrives at the conclusion.
<b>Direct Variation:</b>	Two variables are so related that their ratio remains constant.
<b>Distributive Property:</b>	Multiplication is distributive over addition and subtraction [i.e., $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$ ].
<b>Domain:</b>	The set of all possible values for the unknown in an open sentence.
<b>Equation:</b>	A statement of equality between two mathematical expressions (e.g., $X + 5 = Y - 2$ ).
<b>Equivalent Forms:</b>	Different forms of numbers that name the same number (e.g., fraction, decimal, percent as $\frac{1}{2}$ , .5, 50%).
<b>Expanded Notation:</b>	Involves writing the number in expanded form to show the value of each digit (e.g., $15,629 = 10,000 + 5,000 + 600 + 20 + 9$ ).
<b>Exponent:</b>	A numeral used to tell how many times a number or variable is used as a factor (e.g., $a^2$ , $2^n$ , $y^x$ ).
<b>Exponential Function:</b>	A function whose general equation is $y = a \times b^x$ or $y = a \times b^{kx}$ , where $a$ , $b$ and $k$ stand for constants.
<b>Exponential Notation:</b>	A way of writing numbers using bases and exponents [e.g., $425 = (4 \times 10^2) + (2 \times 10^1) + (5 \times 10^0)$ ].
<b>Expression:</b>	A mathematical phrase that can include operations, numerals and variables. In algebraic terms: $2l + 3x$ ; in numeric terms: $13.4 - 4.7$ .
<b>Factor:</b>	The number or variable multiplied in a multiplication expression.
<b>Factorial:</b>	The expression $n!$ ( $n$ factorial) is the product of all the numbers from 1 to $n$ for any positive integer $n$ .



# ELEMENTARY STANDARDS

## Grades 3-8

### Mathematics

<b>Function:</b>	A relation in which each value of an independent variable is associated with a unique value of the dependent value.
<b>Geoboard:</b>	A board with pegs aligned in grid fashion that permits rubber bands to be wrapped around pegs to form geometric figures.
<b>Graphing Calculator:</b>	A calculator that will store and draw the graphs of several functions at once.
<b>Histogram:</b>	A graphical display that shows frequencies as bars. It shows what proportion of cases fall into each of several categories.
<b>Identity Property:</b>	In addition, adding 0 to a number leaves the number unchanged, or identical ( $2+0=2$ ). Hence, zero is the identity element for addition. In multiplication, the product of any number and 1 is that number ( $4 \times 1=4$ ). Hence, one is the identity element for multiplication.
<b>Independent Events:</b>	Events such that the outcome of the first event has no effect on the probabilities of the outcome of the second event (e.g., two tosses of the same coin are independent events).
<b>Inductive Reasoning:</b>	Generalizations made from particular observations in a common occurrence.
<b>Inequality:</b>	A mathematical sentence that contains a symbol (e.g., $>$ , $<$ , $\geq$ , $\leq$ or $\neq$ ) in which the terms on either side of the symbol are unequal (e.g., $x < y$ , $7 > 3$ , $n \geq 4$ ).
<b>Infinite:</b>	Has no end or goes on forever.
<b>Integer:</b>	A number that is a positive whole number, a negative whole number or zero.
<b>Interquartile Range:</b>	The inter-quartile range is a measure of the spread of or dispersion within a data set. It is calculated by taking the difference between the upper and the lower quartiles.
<b>Inverse:</b>	A new conditional formed by negating both the antecedent and the consequent of a conditional.

ELEMENTARY STANDARDS  
Grades 3-8  
Mathematics

<b>Inverse Function:</b>	<p>Formally, suppose <math>f</math> is a one-to-one function with a domain <math>A</math> and range <math>B</math>. The inverse function <math>f^{-1}</math> is a function with these properties:</p> <ul style="list-style-type: none"><li>▪ <math>f^{-1}</math> has domain <math>B</math> and range <math>A</math>.</li><li>▪ For all <math>x</math> in <math>B</math>, <math>f(f^{-1}(x)) = x</math>, or <math>f \circ f^{-1} = x</math></li></ul> <p>Informally, if <math>f</math> is a function from <math>A</math> to <math>B</math>, then an inverse for <math>f</math> is a function from <math>B</math> to <math>A</math>, with the property that a round trip (a composition) from <math>A</math> to <math>B</math> to <math>A</math> returns each element to itself.</p>
<b>Inverse Operations:</b>	<p>Operations that undo each other (e.g., addition and subtraction are inverse operations; multiplication and division are inverse operations).</p>
<b>Inverse Variation:</b>	<p>When the ratio of one variable to the reciprocal of the other is constant, one of them is said to vary inversely as the other.</p>
<b>Inverse Statement:</b>	<p>Formed by negating the hypothesis and negating the conclusion of the original statement. <i>If <math>p</math>, then <math>q</math></i> becomes <i>if not <math>p</math>, then not <math>q</math></i>.</p>
<b>Irrational Number:</b>	<p>A number that cannot be written as a simple fraction. It is an infinite and non-repeating decimal.</p>
<b>Limit:</b>	<p>A number to which the terms of a sequence get closer so that beyond a certain term all terms are as close as desired to that number.</p>
<b>Line of Best Fit:</b>	<p>The line that fits a set of data points with the smallest value for the sum of the squares of the errors (vertical distances) from the data points to the line; the regression line.</p>
<b>Linear Function:</b>	<p>A function whose general equation is <math>y = mx + b</math>, where <math>m</math> and <math>b</math> stand for constants and <math>m \neq 0</math>.</p>
<b>Linear Measurement:</b>	<p>Measurement in a straight line.</p>

# ELEMENTARY STANDARDS

## Grades 3-8

### Mathematics

<b>Logarithm:</b>	The exponent indicating the power to which a fixed number, the base, must be raised to produce a given number. For example, if $n^x = a$ , the logarithm of a, with n as the base, is x; symbolically, $\log_n a = x$ . If the base is 10, the log of 100 is 2.
<b>Logarithmic Function:</b>	Any function in which an independent variable appears as the argument in the form of a logarithm; they are the inverse functions of exponential functions.
<b>Manipulatives:</b>	Materials that allow students to explore mathematical concepts in a concrete mode.
<b>Mathematical Model:</b>	A representation in the mathematical world of some phenomenon in the real world. It frequently consists of a function or relation specifying how two variables are related.
<b>Matrix:</b>	A rectangular array of numbers representing such things as the coefficients in a system of equations arranged in rows and columns.
<b>Maximum:</b>	The greatest number in a set of data.
<b>Mean:</b>	The sum of the set of numbers divided by n, the number of numbers in the set.
<b>Median:</b>	The number that lies in the middle when a set of numbers is arranged in order. If there are two middle values, the median is the mean of these values.
<b>Metric System:</b>	A system of measurement used throughout the world based on factors of 10. It includes measures of length, weight and capacity.
<b>Minimum:</b>	The least number in a set of data.
<b>Missing Addend:</b>	A member of an addition number sentence in which that term is missing (e.g., $5 + \underline{\quad} = 8$ ).
<b>Mode:</b>	The number(s) that occurs most often in a set of numbers (e.g., in the set 1, 2, 3, 3, 5, 8; the mode is.
<b>Models:</b>	Models are representations of real work phenomena. Numerical tables, graphs, equations or systems of equations are all mathematical representations that may be used

# ELEMENTARY STANDARDS

## Grades 3-8

### Mathematics

to model real world phenomena. A good model should capture the essential character of whatever is being modeled.

<b>Multiple:</b>	A number that is the product of a given integer and another integer (e.g., 6 and 9 are multiples of 3).
<b>Normal Curve:</b>	A graphical plot of a mathematical function (frequency distribution) which is unimodal and symmetrical.
<b><i>One-to-one Correspondence:</i></b>	When one and only one element of a second set is assigned to an element of a first set, all elements of the second set are assigned, and every element of the first set has an assignment, the mapping is called one-to-one (e.g., in the set Bill Clinton, George Bush, Ronald Reagan, Jimmy Carter, Hillary Clinton, Barbara Bush, Nancy Reagan and Rosalynn Carter, there is a one-to-one correspondence between the pairs).
<b>Open Sentence:</b>	A statement that contains at least one unknown. It becomes true or false when a quantity is substituted for the unknown (e.g., $x + 5 = 9$ , $y - 2 = 7$ ).
<b>Order of Operations:</b>	Rules for evaluating an expression: work first within parentheses; then calculate all powers, from left to right; then do multiplications or divisions, from left to right; then do additions and subtractions, from left to right.
<b>Patterns:</b>	Regularities in situations such as those in nature, events, shapes, designs and sets of numbers (e.g., spirals on pineapples, geometric designs in quilts, the number sequence 3, 6, 9, 12,...).
<b>Permutation:</b>	An arrangement of a given number of objects from a given set in which the order of the objects is significant.
<b>Perpendicular Lines:</b>	Two lines that intersect to form right angles (e.g., $\perp$ , $\lrcorner$ , $\llcorner$ ).
<b>Plotting Points:</b>	Locating points by means of coordinates, or a curve by plotted points, representing an equation by means of a curve so constructed.

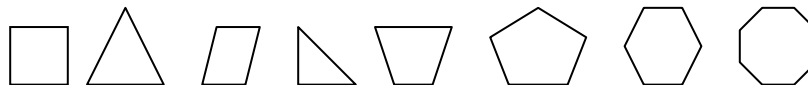
# ELEMENTARY STANDARDS

## Grades 3-8

### Mathematics

**Polygon:**

A union of segments connected end to end, such that each segment intersects exactly two others at its endpoints.

**Powers:**

A number expressed using an exponent. The number  $5^3$  is read five to the third power or five cubed.

**Prime:**

An integer greater than one whose only positive factors are 1 and itself (e.g., 2, 3, 5, 7, 11, 13, 17, and 19).

**Probability:**

A number from 0 to 1 that indicates how likely something is to happen.

**Problem-Solving:**

Finding ways to reach a goal when no routine path is apparent.

**Proof by Contradiction:**

A proof in which, if  $s$  is to be proven, one reasons from not- $s$  until a contradiction is deduced; from this it is concluded that not- $s$  is false, which means that  $s$  is true.

**Proportion:**

An equation of the form  $\frac{a}{b} = \frac{c}{d}$  that states that the two ratios are equivalent.

**Quadrilateral:**

A four-sided polygon.

**Quartiles:**

The three values that divide an ordered set into four subsets of approximately equal size. The second quartile is the median.

**Radian:**

A unit of angular measure equal to  $\frac{1}{2\pi}$  of a complete revolution.

**Range (1):**

The difference between the greatest number and the least number in a set of data.

**Range (2):**

The set of output values for a function.

**Rate of Change:**

The limit of the ratio of an increment of the function value at the point to that of the independent variable as the increment of the variable approaches zero.

# ELEMENTARY STANDARDS

## Grades 3-8

### Mathematics

<b>Ratio:</b>	A comparison of two numbers by division.
<b>Rational numbers:</b>	Any number that can be written in the form $\frac{a}{b}$ where $a$ is any integer and $b$ is any integer except zero.
<b>Real Numbers:</b>	The set consisting of all rational numbers and all irrational numbers.
<b>Reasonableness:</b>	Quality of a solution such that it is not extreme or excessive.
<b>Reciprocal:</b>	The fractional number that results from dividing one by the number.
<b>Rectangular Prism:</b>	A three-dimensional figure whose sides are all rectangles; a box.
<b>Reflection:</b>	A transformation that produces the mirror image of a geometric figure.
<b>Regression:</b>	The line that represents the least deviation from the points in a scatter plot of data.
<b>Regular Polygon:</b>	A polygon in which all sides have the same measure and all angles have the same measure.
<b>Relation:</b>	A set of ordered pairs.
<b>Reliability:</b>	The extent to which a measuring procedure yields the same results on repeated trials.
<b>Repeated Addition:</b>	A model for multiplication (e.g., $2 + 2 + 2 = 3 \times 2$ ).
<b>Rotation:</b>	A transformation that maps every point in the plane to its image by rotating the plane around a fixed point or line.
<b>Scatterplot:</b>	A graph of plotted points that show the relationship between two sets of data.
<b>Scientific Calculator:</b>	A calculator that represents very large or very small numbers in <b>scientific notation</b> with the powering, factorial, square root, negative and reciprocal keys.

# ELEMENTARY STANDARDS

## Grades 3-8

### Mathematics

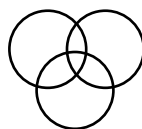
<b>Scientific Notation:</b>	A way in writing a number of terms of an integer power of 10 multiplied by a number greater than or equal to 1 and less than 10.
<b>Sequence:</b>	A set of ordered quantities (e.g., positive integers).
<b>Series:</b>	The indicated sum of the terms of a sequence.
<b>Similarity:</b>	Having the same shape but not necessarily the same size.
<b>Simple Event:</b>	An event whose probability can be obtained from consideration of a single occurrence (e.g., the tossing of a coin is a simple event).
<b>Simulation:</b>	Modeling a real event without actually observing the event.
<b>Slope:</b>	The slope of a line is the ratio of the change in y to the corresponding change in x; the constant m in the linear function equation; rise/run.
<b>Standard Deviation:</b>	The square root of the variance.
<b>Stem-and-Leaf Plot:</b>	<p>A frequency distribution made by arranging data (e.g., student scores on a test were 98, 96, 85, 93, 83, 87, 85, 87, 93, 75, 77 and 83). This data is displayed in a stem-and-leaf plot below.</p> <div><div>9</div><div>8</div><div>7</div><div>8, 6, 3, 3</div><div>7, 7, 5, 5, 3, 3</div><div>7, 5</div></div>
<b>Systems of Equations:</b>	Two or more equations that are conditions imposed simultaneously on all the variables, but may or may not have common solutions (e.g., $x + y = 2$ , and $3x + 2y = 5$ ).
<b>Symmetry:</b>	A line of symmetry separates a figure into two congruent halves, each of which is a reflection of the other (e.g., $\emptyset$ , the line through the center of the circle divides it into congruent halves).
<b>T-Test:</b>	A statistical test done to test the difference of means of two samples.

# ELEMENTARY STANDARDS

## Grades 3-8

### Mathematics

<b>Tessellation:</b>	A repetitive pattern of polygons that covers an area with no holes and no overlaps (e.g., floor tiles).
<b>Transformation:</b>	An operation on a geometric figure by which each point gives rise to a unique image.
<b>Translation:</b>	A transformation that moves a geometric figure by sliding each of the points the same distance in the same direction.
<b>Tree Diagram:</b>	A diagram used to show the total number of possible outcomes in a probability experiment.
<b>Trigonometric Functions:</b>	A function (e.g., sine, cosine, tangent, cotangent, secant, cosecant) whose independent variable is an angle measure, usually in degrees or radians.
<b>Valid Argument:</b>	An argument with the property that no matter what statements are substituted in the premises, the truth value of the form is true. If the premises are true, then the conclusion is true.
<b>Variable:</b>	A symbol used to stand for any one of a given set of numbers or other objects (e.g., in the equation $y = x + 5$ , $y$ and $x$ are variables).
<b>Variance:</b>	In a data set, the sum of the squared deviations divided by one less than the number of elements in the set (sample variance $s^2$ ) or by the number of elements in the set (population variance $\sigma^2$ ).
<b>Vector:</b>	A quantity that has both magnitude and direction (e.g., physical quantities such as velocity and force).
<b>Venn Diagram:</b>	A display that pictures unions and intersections of sets.





ELEMENTARY STANDARDS  
Grades 3-8  
Mathematics

<b>Volume:</b>	The amount of space enclosed in a space (3-dimensional) figure, measured in cubic units.
<b>Y-Intercept:</b>	The y-intercept of a line is the y-coordinate of the point at which the graph of an equation crosses the y-axis.
<b><math>\pi</math>:</b>	pi, the ratio of the circumference of a circle to its diameter: 3.1415926535.